Economics Letters 211 (2022) 110233

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Separable rules to share the revenues from broadcasting sports leagues☆

Gustavo Bergantiños [a], Juan D. Moreno-Ternero [b,*]

[a] ECOBAS, Universidade de Vigo, ECOSOT, 36310, Vigo, Spain
[b] Universidad Pablo de Olavide, Department of Economics, Carretera de Utrera, Km. 1, 41013, Seville, Spain

ARTICLE INFO

Article history:
Received 22 September 2021
Received in revised form 26 November 2021
Accepted 11 December 2021
Available online 23 December 2021

JEL classification:
D63
C71
Z20

Keywords:
Resource allocation
Broadcasting problems
Additivity
Equal treatment of equals
Monotonicity

ABSTRACT

We characterize a large family of rules for the problem of sharing the revenues from broadcasting sports leagues, by combining just two basic axioms: *additivity* and a weak form of *equal treatment of equals*. We also explore the implications of the principle of *monotonicity* for the resulting family of *separable* rules. Based on it, we derive new characterization results for focal members of the family.

1. Introduction

Resource allocation problems are ubiquitous in Economics. In this note, we consider the problem of sharing the revenues from broadcasting sports leagues among participating teams, based on the audiences they generate, recently introduced in Bergantiños and Moreno-Ternero (2020a). Our starting point is to explore the implications of the combination of two basic axioms: *additivity* and *weak equal treatment of equals*. The former is a standard condition in axiomatic work, which can be traced back to Shapley (1953). The latter is the weakest possible version of impartiality, a fundamental principle in the theory of justice (e.g., Moreno-Ternero and Roemer, 2006). We show that the combination of those two axioms characterizes a large family of *separable* rules, in which the audience of each game is allocated among teams according to three rates (referring to the home team, the away team, and all other teams, respectively). We then explore the implications of the principle of *monotonicity* (e.g., Thomson and Myerson, 1980) for such a family of *separable* rules. This analysis allows us to derive new characterizations for focal members of the family (such as *split* rules and *compromise* rules) when combined with some other basic axioms in this model.

2. The model

We consider the model introduced by Bergantiños and Moreno-Ternero (2020a).[1] Let N describe a (fixed) finite set of teams, with cardinality $n \geq 3$. For each pair $i, j \in N$, let $a_{ij} \geq 0$ be the broadcasting audience for the game played by i and j at i's stadium. Let $A \in \mathcal{A}_{n\times n}$ denote the resulting matrix of broadcasting audiences in the (double round-robin) tournament. Each matrix with zero entries in the diagonal (and non-negative entries in the rest of the matrix) represents a **problem**, whose set is denoted $\mathcal{P}$. For each $A \in \mathcal{P}$, let $\|A\| = \sum_{i,j\in N} a_{ij}$. A **rule** is a mapping $R : \mathcal{P} \to \mathbb{R}^n$ such that, for each $A \in \mathcal{P}$, $\sum_{i\in N} R_i(A) = \|A\|$. We consider the following family of rules:

***Separable* rules** $\{G^{xyz}\}$. Let $x, y, z \in \mathcal{A}_{n\times n}$ (with zero entries in the diagonals) satisfy

☆ We thank Joseph E. Harrington (editor of this journal) and an anonymous referee for helpful comments and suggestions. Financial support from the Ministerio de Ciencia e Innovación, Spain through the research projects ECO2017-82241-R, ECO2017-83069-P, PID2020-115011GB-I00, and PID2020-113440GB-I00 (funded by MCIN/AEI/ 10.13039/501100011033/), Xunta de Galicia, Spain through grant ED431B 2019/34 and Junta de Andalucía, Spain through grants P18-FR-2933, and A-SEJ-14-UGR20 is gratefully acknowledged. Funding for open access publishing: Universidad Pablo de Olavide/CBUA.

* Corresponding author.
E-mail address: jdmoreno@upo.es (J.D. Moreno-Ternero).

[1] See also Bergantiños and Moreno-Ternero (2020b, 2021a,b, 2022).

https://doi.org/10.1016/j.econlet.2021.110233

- for each pair $i, j \in N$, $i \neq j$,

$x_{ij} + y_{ij} + (n-2)\, z_{ij} = 1,$

- for each trio $i, j, k \in N$, $i \neq j \neq k$,

$$y_{ij} + z_{ik} = z_{ij} + y_{ik},$$
$$x_{ji} + z_{ki} = z_{ji} + x_{ki},$$
$$x_{ij} + y_{ji} = y_{ij} + x_{ji}.$$

Then, for each $A \in \mathcal{P}$, and each $i \in N$,

$$G_i^{xyz}(A) = \sum_{j \in N\setminus\{i\}} x_{ij} a_{ij} + \sum_{j \in N\setminus\{i\}} y_{ji} a_{ji} + \sum_{j,k \in N\setminus\{i\}} z_{jk} a_{jk}.$$

In words, each separable rule (G^{xyz}) allocates the audience of each game (a_{ij}) according to three rates. The home team (i) receives x_{ij}. The away team (j) gets y_{ij}. Other teams get z_{ij}.

If $x_{ij} = 1-\lambda$, $y_{ij} = \lambda$, and $z_{ij} = 0$, for each pair $i, j \in N$, $i \neq j$, we obtain a focal family:

Split rules $\left\{S^{\lambda}\right\}_{\lambda \in [0,1]}$: for each $\lambda \in [0, 1]$, each $A \in \mathcal{P}$, and each $i \in N$,

$$S_i^{\lambda}(A) = (1-\lambda) \sum_{j \in N\setminus\{i\}} a_{ij} + \lambda \sum_{j \in N\setminus\{i\}} a_{ji}.$$

If $x_{ij} = y_{ij} = \frac{1}{2}$ and $z_{ij} = 0$, for each pair $i, j \in N$, $i \neq j$, we obtain the focal *equal-split* rule. Two other focal rules, known as the *uniform* rule and *concede-and-divide*, are obtained when for each pair $i, j \in N$, $i \neq j$, $x_{ij} = y_{ij} = z_{ij} = \frac{1}{n}$, and $x_{ij} = y_{ij} = \frac{n-1}{n-2}$ and $z_{ij} = \frac{-1}{n-2}$, respectively. Thus, the so-called generalized *compromise* rules arising from all linear combinations of those rules belong to the family of *separable* rules too. Formally,

Generalized compromise rules $\left\{GUC^{\lambda}\right\}_{\lambda \in \mathbb{R}}$: for each $\lambda \in \mathbb{R}$, each $A \in \mathcal{P}$, and each $i \in N$,

$$GUC_i^{\lambda}(A) = (1-\lambda)\frac{||A||}{n} + \lambda \frac{(n-1)\sum_{j \in N}(a_{ij} + a_{ji}) - ||A||}{n-2}$$

We now introduce axioms. The first says that if two teams have the same audiences, when facing a third, and each other, they receive the same amount.

Weak equal treatment of equals: For each $A \in \mathcal{P}$, and each pair $i, j \in N$ such that $a_{ij} = a_{ji}$, $a_{ik} = a_{jk}$, and $a_{ki} = a_{kj}$, for each $k \in N \setminus \{i, j\}$,

$$R_i(A) = R_j(A).$$

The second says that revenues are additive on A. An interpretation is that the aggregation of the revenue sharing in two seasons (involving the same teams) is equivalent to the revenue sharing in the hypothetical combined season aggregating the audiences of the corresponding games in both seasons.

Additivity: For each pair A and $A' \in \mathcal{P}$,

$$R\left(A + A'\right) = R(A) + R\left(A'\right).$$

The next says that an audience redistribution of the games involving a pair of teams does not affect the revenues they obtain.

Pairwise reallocation proofness: For each pair $A, A' \in \mathcal{P}$, and each pair $i_0, j_0 \in N$, such that $a_{ij} = a'_{ij}$, for each pair $\{i, j\} \neq \{i_0, j_0\}$, and $a_{i_0 j_0} + a_{j_0 i_0} = a'_{i_0 j_0} + a'_{j_0 i_0}$,

$$R_k(A) = R_k(A') \text{ for each } k = i_0, j_0.$$

Somewhat related, if only games played by two teams have positive audience, the total audience should be allocated to them.

Stand-alone pair: For each $A \in \mathcal{P}$ and each pair $i, j \in N$ such that $a_{kl} = 0$ for each pair $\{k, l\} \in N$ with $(k, l) \neq (i, j)$ and $(k, l) \neq (j, i)$,

$$R_i(A) + R_j(A) = ||A||.$$

A strengthening says that if a team has null audience, then it gets no revenue.

Null team: For each $A \in \mathcal{P}$, and each $i \in N$, such that for each $j \in N$, $a_{ij} = 0 = a_{ji}$,

$$R_i(A) = 0.$$

We conclude introducing several *monotonicity* axioms.[2]

Aggregate monotonicity: For each pair A and $A' \in \mathcal{P}$ and each $i \in N$,

$$||A|| \le \left|\left|A'\right|\right| \quad \Rightarrow \quad R_i(A) \le R_i\left(A'\right).$$

Overall monotonicity: For each pair A and $A' \in \mathcal{P}$ and each $i \in N$,

$$a_{jk} \le a'_{jk} \text{ for each pair } j, k \in N \quad \Rightarrow \quad R_i(A) \le R_i\left(A'\right).$$

Pairwise monotonicity: For each pair A and $A' \in \mathcal{P}$ and each $i \in N$,

$$a_{kj} + a_{jk} \le a'_{kj} + a'_{jk} \text{ for each pair } j, k \in N \quad \Rightarrow \quad R_i(A) \le R_i(A').$$

Team monotonicity: For each pair A and $A' \in \mathcal{P}$ and each $i \in N$,

$$\left.\begin{array}{l} a_{ij} \le a'_{ij} \text{ for each } j \in N \setminus \{i\} \text{ and} \\ a_{ji} \le a'_{ji} \text{ for each } j \in N \setminus \{i\} \end{array}\right\} \quad \Rightarrow \quad R_i(A) \le R_i\left(A'\right).$$

Weak team monotonicity: For each pair A and $A' \in \mathcal{P}$ and each $i \in N$,

$$\left.\begin{array}{l} a_{ij} \le a'_{ij} \text{ for each } j \in N \setminus \{i\} \text{ and} \\ a_{ji} \le a'_{ji} \text{ for each } j \in N \setminus \{i\} \\ a_{jk} = a'_{jk} \text{ when } i \notin \{j, k\} \end{array}\right\} \quad \Rightarrow \quad R_i(A) \le R_i\left(A'\right).$$

Reciprocal monotonicity: For each pair A and $A' \in \mathcal{P}$ and each $i \in N$,

$$\left.\begin{array}{l} a_{ij} = a'_{ij} \text{ for each } j \in N \setminus \{i\} \text{ and} \\ a_{ji} = a'_{ji} \text{ for each } j \in N \setminus \{i\} \\ a_{jk} \le a'_{jk} \text{ when } i \notin \{j, k\} \end{array}\right\} \quad \Rightarrow \quad R_i(A) \ge R_i\left(A'\right).$$

3. Characterization results

Our main result states that the *separable* rules are characterized by the first two axioms introduced above.

Theorem 1. *A rule satisfies weak equal treatment of equals and additivity if and only if it is a separable rule.*

The next result gathers the effect that monotonicity axioms have on separable rules.

Proposition 1. *The following statements hold:*

- *A separable rule S^{xyz} satisfies weak team monotonicity if and only if $x_{ij} \ge 0$ and $y_{ij} \ge 0$, for each pair $i, j \in N$, $i \neq j$.*
- *A separable rule S^{xyz} satisfies overall monotonicity if and only if $x_{ij} \ge 0$, $y_{ij} \ge 0$, and $z_{ij} \ge 0$, for each pair $i, j \in N$, $i \neq j$.*

[2] The reader is referred to Bergantiños and Moreno-Ternero (2022) for verbal descriptions of these axioms. Similar axioms have also been recently explored in related models (e.g., Calleja et al., 2020; Juarez et al., 2020).

- *A separable rule S^{xyz} satisfies pairwise monotonicity if and only if $x_{ij} = y_{ji} \geq 0$, $y_{ij} = x_{ji} \geq 0$, and $z_{ij} = z_{ji} \geq 0$, for each pair $i, j \in N$, $i \neq j$.*
- *A separable rule S^{xyz} satisfies reciprocal monotonicity if and only if $z_{ij} \leq 0$, for each pair $i, j \in N$, $i \neq j$.*

Two monotonicity axioms have been excluded from the previous result because they offer more interesting implications, gathered in the next two results.

Theorem 2. *A rule satisfies weak equal treatment of equals, additivity and team monotonicity if and only if it is a split rule.*

Theorem 3. *A rule satisfies weak equal treatment of equals and aggregate monotonicity if and only if it is the uniform rule.*

Somewhat surprisingly, adding *stand-alone pair* (or *null team*) to each of the first three statements of Proposition 1, we obtain the same effect.[3]

Theorem 4. *A rule satisfies weak equal treatment of equals, additivity, stand-alone pair or null team, and weak team monotonicity, overall monotonicity or pairwise monotonicity, if and only if it is a split rule.*

If we replace *stand alone pair* (or *null team*) by *pairwise reallocation proofness* we obtain completely different results.

Theorem 5. *A rule R satisfies weak equal treatment of equals, additivity, pairwise reallocation proofness, and*

- *weak team monotonicity if and only if $R \in \left\{GUC^{\lambda} : \lambda \geq -\frac{1}{n-1}\right\}$.*
- *overall monotonicity or pairwise monotonicity, if and only if $R \in \left\{GUC^{\lambda} : -\frac{1}{n-1} \leq \lambda \leq \frac{n-2}{2(n-1)}\right\}$.*
- *reciprocal monotonicity if and only if $R \in \left\{GUC^{\lambda} : \lambda \geq \frac{n-2}{2(n-1)}\right\}$.*
- *team monotonicity if and only if it is the equal-split rule.*[4]

Appendix. Proof of the results

Proof of Theorem 1. It is straightforward to show that each *separable* rule satisfies *additivity*. As for *weak equal treatment of equals*, let i and j be as in its definition. Then,

$$\begin{aligned}G_i^{xyz}(A) &= \sum_{k\in N\setminus\{i\}} x_{ik}a_{ik} + \sum_{k\in N\setminus\{i\}} y_{ki}a_{ki} + \sum_{k,l\in N\setminus\{i\}} z_{kl}a_{kl} \\ &= x_{ij}a_{ij} + y_{ji}a_{ji} + \sum_{k\in N\setminus\{i,j\}} \left(x_{ik}a_{ik} + z_{jk}a_{jk}\right) \\ &\quad + \sum_{k\in N\setminus\{i,j\}} \left(y_{ki}a_{ki} + z_{kj}a_{kj}\right) + \sum_{k,l\in N\setminus\{i,j\}} z_{kl}a_{kl}.\end{aligned}$$

As $a_{ij} = a_{ji}$, $a_{ik} = a_{jk}$ and $a_{ki} = a_{kj}$, for each $k \in N\setminus\{i,j\}$, the conditions on x, y, z yield

$$\begin{aligned}x_{ij}a_{ij} + y_{ji}a_{ji} &= y_{ij}a_{ij} + x_{ji}a_{ji} \\ x_{ik}a_{ik} + z_{jk}a_{jk} &= z_{ik}a_{ik} + x_{jk}a_{jk}, \text{ and} \\ y_{ki}a_{ki} + z_{kj}a_{kj} &= z_{ki}a_{ki} + y_{kj}a_{kj}.\end{aligned}$$

Then,

$$\begin{aligned}G_i^{xyz}(A) &= y_{ij}a_{ij} + x_{ji}a_{ji} + \sum_{k\in N\setminus\{i,j\}} \left(z_{ik}a_{ik} + x_{jk}a_{jk}\right) \\ &\quad + \sum_{k\in N\setminus\{i,j\}} \left(z_{ki}a_{ki} + y_{kj}a_{kj}\right) + \sum_{k,l\in N\setminus\{i,j\}} z_{kl}a_{kl} \\ &= \sum_{k\in N\setminus\{j\}} x_{jk}a_{jk} + \sum_{k\in N\setminus\{j\}} y_{kj}a_{kj} + \sum_{k,l\in N\setminus\{j\}} z_{kl}a_{kl} = G_j^{xyz}(A).\end{aligned}$$

Conversely, let R be a rule satisfying both axioms. Let $i, j \in N$, $i \neq j$, and $\mathbf{1}^{ij}$ be the matrix with zero entries, except for $a_{ij} = 1$. By *weak equal treatment of equals*, there exists $z_{ij} \in \mathbb{R}$ such that $R_k\left(\mathbf{1}^{ij}\right) = z_{ij}$ for each $k \in N\setminus\{i,j\}$. Let $R_i\left(\mathbf{1}^{ij}\right) = x_{ij}$ and $R_j\left(\mathbf{1}^{ij}\right) = y_{ij}$. Then,

$$1 = \|\mathbf{1}^{ij}\| = \sum_{k\in N} R_k\left(\mathbf{1}^{ij}\right) = x_{ij} + y_{ij} + (n-2)z_{ij}.$$

Let $i, j, k \in N$, $i \neq j \neq k$. By *additivity* and *weak equal treatment of equals*,

$$\begin{aligned}y_{ij} + z_{ik} &= R_j\left(\mathbf{1}^{ij} + \mathbf{1}^{ik}\right) = R_k\left(\mathbf{1}^{ij} + \mathbf{1}^{ik}\right) = z_{ij} + y_{ik}, \\ x_{ji} + z_{ki} &= R_j\left(\mathbf{1}^{ji} + \mathbf{1}^{ki}\right) = R_k\left(\mathbf{1}^{ji} + \mathbf{1}^{ki}\right) = z_{ji} + x_{ki}, \text{ and} \\ x_{ij} + y_{ji} &= R_i\left(\mathbf{1}^{ij} + \mathbf{1}^{ji}\right) = R_j\left(\mathbf{1}^{ij} + \mathbf{1}^{ji}\right) = y_{ij} + x_{ji}.\end{aligned}$$

For each $i \in N$, let $x_{ii} = y_{ii} = z_{ii} = 0$. Let $x = \left(x_{ij}\right)_{i,j\in N}$, $y = \left(y_{ij}\right)_{i,j\in N}$, $z = \left(z_{ij}\right)_{i,j\in N}$. By *additivity*, we can deduce from the above conditions that $R(A) = S^{xyz}(A)$, for each $A \in \mathcal{P}$. ■

Proof of Proposition 1. We only focus on the non-trivial implications. Let $\mathbf{0}$ denote the matrix where all audiences are 0. By *weak equal treatment of equals*, $R_i(\mathbf{0}) = R_j(\mathbf{0})$, for each pair $i, j \in N$. As $\|\mathbf{0}\| = 0$, it follows that $R_i(\mathbf{0}) = 0$ for each $i \in N$.

Let S^{xyz} be a *separable* rule satisfying *weak team monotonicity*. Thus, for each pair $i, j \in N$, $i \neq j$, $R_i\left(\mathbf{1}^{ij}\right) = x_{ij}$, $R_j\left(\mathbf{1}^{ij}\right) = y_{ij}$, and $R_k\left(\mathbf{1}^{ij}\right) = z_{ij}$ for each $k \in N\setminus\{i,j\}$. By *weak team monotonicity*, $x_{ij} = R_i\left(\mathbf{1}^{ij}\right) \geq R_i(\mathbf{0}) = 0$ and $y_{ij} = R_j\left(\mathbf{1}^{ij}\right) \geq R_j(\mathbf{0}) = 0$, for each pair $i, j \in N$, $i \neq j$, which proves the first statement.

Let S^{xyz} be a *separable* rule satisfying *overall monotonicity*. Thus, for each pair $i, j \in N$, $i \neq j$, $R_i\left(\mathbf{1}^{ij}\right) = x_{ij}$, $R_j\left(\mathbf{1}^{ij}\right) = y_{ij}$ and $R_k\left(\mathbf{1}^{ij}\right) = z_{ij}$ for each $k \in N\setminus\{i,j\}$. By *overall monotonicity*, $x_{ij} = R_i\left(\mathbf{1}^{ij}\right) \geq R_i(\mathbf{0}) = 0$ and $y_{ij} = R_j\left(\mathbf{1}^{ij}\right) \geq R_j(\mathbf{0}) = 0$, for each pair $i, j \in N$, $i \neq j$. Moreover, $z_{ij} = R_k\left(\mathbf{1}^{ij}\right) \geq R_k(\mathbf{0}) = 0$, for each $k \in N\setminus\{i,j\}$, which proves the second statement.

As for *pairwise monotonicity*, let A, A' and i be as in its definition. Then,

$$\begin{aligned}G_i^{xyz}(A) &= \sum_{j\in N\setminus\{i\}} x_{ij}a_{ij} + \sum_{j\in N\setminus\{i\}} y_{ji}a_{ji} + \sum_{j,k\in N\setminus\{i\}} z_{jk}a_{jk} \\ &= \sum_{j\in N\setminus\{i\}} \left(x_{ij}a_{ij} + y_{ji}a_{ji}\right) + \sum_{j,k\in N\setminus\{i\}, j\leq k} \left(z_{jk}a_{jk} + z_{kj}a_{kj}\right).\end{aligned}$$

By the condition at the statement,

$$\begin{aligned}x_{ij}a_{ij} + y_{ji}a_{ji} &\leq x_{ij}a'_{ij} + y_{ji}a'_{ji} \text{ for each } j \in N\setminus\{i\} \text{ and} \\ z_{jk}a_{jk} + z_{kj}a_{kj} &\leq z_{jk}a'_{jk} + z_{kj}a'_{kj} \text{ for each pair } j, k \in N\setminus\{i\}.\end{aligned}$$

Then,

$$\begin{aligned}G_i^{xyz}(A) &\leq \sum_{j\in N\setminus\{i\}} \left(x_{ij}a'_{ij} + y_{ji}a'_{ji}\right) + \sum_{j,k\in N\setminus\{i\}, j\leq k} \left(z_{jk}a'_{jk} + z_{kj}a'_{kj}\right) \\ &= G_i^{xyz}(A').\end{aligned}$$

Now, let S^{xyz} be a *separable* rule satisfying *pairwise monotonicity*. Thus, for each pair $i, j \in N$, $i \neq j$, $R_i\left(\mathbf{1}^{ij}\right) = x_{ij}$, $R_j\left(\mathbf{1}^{ij}\right) = y_{ij}$ and, for each $k \in N\setminus\{i,j\}$, $R_k\left(\mathbf{1}^{ij}\right) = z_{ij}$. By *pairwise monotonicity*, $x_{ij} = R_i\left(\mathbf{1}^{ij}\right) \geq R_i(\mathbf{0}) = 0$, $y_{ij} = R_j\left(\mathbf{1}^{ij}\right) \geq R_j(\mathbf{0}) = 0$, for each

3 As for the fourth statement, *reciprocal monotonicity* is implied by the combination of those axioms.

4 Additivity is not needed for this result.

pair $i, j \in N$, $i \neq j$. Moreover, $z_{ij} = R_k\left(\mathbf{1}^{ij}\right) \geq R_k(\mathbf{0}) = 0$, for each $k \in N \setminus \{i, j\}$. Furthermore, $R\left(\mathbf{1}^{ij}\right) = R\left(\mathbf{1}^{ji}\right)$. Thus, $x_{ij} = y_{ji}$, $y_{ij} = x_{ji}$, and $z_{ij} = z_{ji}$, which proves the third statement.

As for *reciprocal monotonicity*, let A, A' and i be as in the definition of the axiom. Then,

$$G_i^{xyz}(A) = \sum_{j \in N \setminus \{i\}} x_{ij} a_{ij} + \sum_{j \in N \setminus \{i\}} y_{ji} a_{ji} + \sum_{j,k \in N \setminus \{i\}} z_{jk} a_{jk}.$$

Let $j, k \in N \setminus \{i\}$. By the condition at the statement, $z_{jk} \leq 0$. Then,

$$G_i^{xyz}(A) \geq \sum_{j \in N \setminus \{i\}} x_{ij} a'_{ij} + \sum_{j \in N \setminus \{i\}} y_{ji} a'_{ji} + \sum_{j,k \in N \setminus \{i\}} z_{jk} a'_{jk} = G_i^{xyz}\left(A'\right).$$

Now, let S^{xyz} be a *separable* rule satisfying *reciprocal monotonicity*. Thus, for each pair $i, j \in N$, $i \neq j$, $R_i\left(\mathbf{1}^{ij}\right) = x_{ij}$, $R_j\left(\mathbf{1}^{ij}\right) = y_{ij}$ and, for each $k \in N \setminus \{i, j\}$, $R_k\left(\mathbf{1}^{ij}\right) = z_{ij}$. By *reciprocal monotonicity*, $z_{ij} = R_k\left(\mathbf{1}^{ij}\right) \leq R_k(\mathbf{0}) = 0$, for each pair $i, j \in N$, $i \neq j$, and each $k \in N \setminus \{i, j\}$, which proves the fourth statement. ■

Proof of Theorem 2. It is straightforward to show that each *split* rule satisfies the axioms at the statement. Conversely, let R be a rule satisfying the three axioms. By Theorem 1, we know that R is a *separable* rule, i.e., $R = S^{xyz}$ for a trio x, y and z defined as in the proof of Theorem 1. Thus, for each pair $i, j \in N$, $i \neq j$, and each $k \in N \setminus \{i, j\}$, $R_i\left(\mathbf{1}^{ij}\right) = x_{ij}$, $R_j\left(\mathbf{1}^{ij}\right) = y_{ij}$ and $R_k\left(\mathbf{1}^{ij}\right) = z_{ij}$. Let A, A', and $i \in N$ be such that for each $j \in N \setminus \{i\}$, $a_{ij} = a'_{ij}$ and $a_{ji} = a'_{ji}$. By *team monotonicity*, $R_i(A) = R_i\left(A'\right)$. By *weak equal treatment of equals*, $R_i(\mathbf{0}) = 0$ for each $i \in N$. Now, for each pair $i, j \in N$, $i \neq j$ and each $k \in N \setminus \{i, j\}$, $z_{ij} = R_k\left(\mathbf{1}^{ij}\right) = R_k(\mathbf{0}) = 0$. Using arguments similar to those in the proof of Bergantiños and Moreno-Ternero (2021b) we can find $\lambda \in \mathbb{R}$ such that for each pair $i, j \in N$, $i \neq j$, $x_{ij} = 1 - \lambda$ and $y_{ij} = \lambda$. Let $A \in \mathcal{P}$ and $i \in N$. By *additivity*,

$$\begin{aligned} R_i(A) &= \sum_{j,k \in N} a_{jk} R_i\left(\mathbf{1}^{jk}\right) = \sum_{j \in N \setminus \{i\}} a_{ij} R_i\left(\mathbf{1}^{ij}\right) + \sum_{j \in N \setminus \{i\}} a_{ji} R_i\left(\mathbf{1}^{ji}\right) \\ &= (1 - \lambda) \sum_{j \in N \setminus \{i\}} a_{ij} + \lambda \sum_{j \in N \setminus \{i\}} a_{ji}. \end{aligned}$$

By *team monotonicity*, $1 - \lambda = x_{ij} = R_i\left(\mathbf{1}^{ij}\right) \geq R_i(\mathbf{0}) = 0$ and $\lambda = y_{ij} = R_j\left(\mathbf{1}^{ij}\right) \geq R_j(\mathbf{0}) = 0$. Thus, $\lambda \in [0, 1]$ and hence R is a *split* rule. ■

Proof of Theorem 4. We know from Theorem 1 that R is a *separable* rule, i.e., $R = S^{xyz}$ for a trio x, y, and z, defined as in the corresponding statement. Let $i, j \in N$, $i \neq j$. By *stand alone pair*, $1 = R_i\left(\mathbf{1}^{ij}\right) + R_j\left(\mathbf{1}^{ij}\right) = x_{ij} + y_{ij}$. As, by definition of separable rule, $x_{ij} + y_{ij} + (n-2) z_{ij} = 1$, it follows that $z_{ij} = 0$. Then, as in the proof of Theorem 2, we obtain that there exists $\lambda \in \mathbb{R}$, such that, for each $A \in \mathcal{P}$ and each $i \in N$, $R_i(A) = (1 - \lambda) \sum_{j \in N \setminus \{i\}} a_{ij} + \lambda \sum_{j \in N \setminus \{i\}} a_{ji}$. Note that $1 - \lambda = R_i\left(\mathbf{1}^{ij}\right)$ and $\lambda = R_j\left(\mathbf{1}^{ij}\right)$, whereas $R_i(\mathbf{0}) = R_j(\mathbf{0}) = 0$. Thus, by *weak team monotonicity*, *overall monotonicity*, or *pairwise monotonicity*, we obtain that $0 \leq \lambda \leq 1$, from where we conclude that R is a *split* rule. ■

The remaining results are refinements or restatements of results in Bergantiños and Moreno-Ternero (2021b, 2022) and the proofs can simply be replicated here with slight modifications.

References

Bergantiños, G., Moreno-Ternero, J.D., 2020a. Sharing the revenues from broadcasting sport events. Manage. Sci. 66 (6), 2417–2431.

Bergantiños, G., Moreno-Ternero, J.D., 2020b. Allocating extra revenues from broadcasting sports leagues. J. Math. Econom. 90, 65–73.

Bergantiños, G., Moreno-Ternero, J.D., 2021a. Compromising to share the revenues from broadcasting sports leagues. J. Econ. Behav. Organ. 183, 57–74.

Bergantiños, G., Moreno-Ternero, J.D., 2021b. On the axiomatic approach to sharing the revenues from broadcasting sports leagues. Soc. Choice Welf. http://dx.doi.org/10.1007/s00355-021-01360-9, Forthcoming.

Bergantiños, G., Moreno-Ternero, J.D., 2022. Monotonicity in sharing the revenues from broadcasting sports leagues. European J. Oper. Res. 297, 338–346.

Calleja, P., Llerena, F., Sudholter, P., 2020. Monotonicity and weighted prenucleoli: A characterization without consistency. Math. Oper. Res. 45, 1056–1068.

Juarez, R., Nitta, K., Vargas, M., 2020. Profit-sharing and efficient time allocation. Econom. Theory 70, 817–846.

Moreno-Ternero, J., Roemer, J., 2006. Impartiality, priority and solidarity in the theory of justice. Econometrica 74, 1419–1427.

Shapley, L., 1953. A value for n-person games. In: Kuhn, H.W., Tucker, A.W. (Eds.), Contributions to the Theory of Games II (Annals of Mathematics Studies 28). Princeton University Press, Princeton, pp. 307–317.

Thomson, W., Myerson, R., 1980. Monotonicity and independence axioms. Internat. J. Game Theory 9, 37–49.